

Logarithms and Log Properties

Definition

$$y = \log_b x \text{ is equivalent to } x = b^y$$

Example

$$\log_5 125 = 3 \text{ because } 5^3 = 125$$

Special Logarithms

$$\ln x = \log_e x \quad \text{natural log}$$

$$\log x = \log_{10} x \quad \text{common log}$$

where $e = 2.718281828\mathbf{K}$

Logarithm Properties

$$\log_b b = 1 \quad \log_b 1 = 0$$

$$\log_b b^x = x \quad b^{\log_b x} = x$$

$$\log_b (x^r) = r \log_b x$$

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y} \right) = \log_b x - \log_b y$$

The domain of $\log_b x$ is $x > 0$

Factoring and Solving

Factoring Formulas

$$x^2 - a^2 = (x+a)(x-a)$$

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x-a)^2$$

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$$

$$x^3 - 3ax^2 + 3a^2x - a^3 = (x-a)^3$$

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$

$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

If n is odd then,

$$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + \mathbf{L} + a^{n-1})$$

$$x^n + a^n$$

$$= (x+a)(x^{n-1} - ax^{n-2} + a^2x^{n-3} - \mathbf{L} + a^{n-1})$$

Quadratic Formula

Solve $ax^2 + bx + c = 0$, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ - Two real unequal solns.

If $b^2 - 4ac = 0$ - Repeated real solution.

If $b^2 - 4ac < 0$ - Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \text{ or } p > b$$

Completing the Square

$$\text{Solve } 2x^2 - 6x - 10 = 0$$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of x , square it and add it to both sides

$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2 = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs

Constant Function

$$y = a \quad \text{or} \quad f(x) = a$$

Graph is a horizontal line passing through the point $(0, a)$.

Line/Linear Function

$$y = mx + b \quad \text{or} \quad f(x) = mx + b$$

Graph is a line with point $(0, b)$ and slope m .

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept $(0, b)$ is

$$y = mx + b$$

Point – Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m(x - x_1)$$

Parabola/Quadratic Function

$$y = a(x - h)^2 + k \quad f(x) = a(x - h)^2 + k$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k) .

Parabola/Quadratic Function

$$y = ax^2 + bx + c \quad f(x) = ax^2 + bx + c$$

The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Parabola/Quadratic Function

$$x = ay^2 + by + c \quad g(y) = ay^2 + by + c$$

The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex

$$\text{at} \left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a} \right).$$

Circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius r and center (h, k) .

Ellipse

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k) , vertices a units left/right of center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.

Hyperbola

$$\frac{(y - k)^2}{b^2} - \frac{(x - h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k) , vertices b units up/down from the center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.